

Tariff Design with Varying Degrees of Commitment

Maria Cristina T. Terra*

Graduate School of Economics – Fundação Getulio Vargas

Abstract:

This paper provides a novel perspective on the dynamics of infant industry protection. Trade policies are analyzed when the industrial sector generates positive externalities in production, and there are adjustment costs to changing production between sectors. If the government is able to precommit to its future tariff schedule, the welfare maximizing policy is to maintain a positive tariff forever, even after the steady state is reached. However, if no precommitment is possible, the only time-consistent policy is zero tariff always. The case with precommitment for a limited period of time is also analyzed.

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* Address:

Graduate School of Economics
Fundação Getulio Vargas
Praia de Botafogo 190 sala 1125
22253-900 - Rio de Janeiro, RJ, Brazil
Tel: (5521) 536-9243 Fax: (5521) 536-9450
E-mail: terra@fgv.br

1. Introduction

The infant industry argument for protection has been in the literature since the 18th Century. The basic classical argument states that when the private marginal productivity of factors of production in the industrial sector is below the social rate due to some externality, factor allocation to that sector is lower than optimal. If industrialization eliminates the distortion, infant industry protection may be welfare-enhancing.

This paper focuses on the implications of the government's commitment power for the design of tariff policy.¹ The main new result is that, under the conditions that the earlier literature claimed would require infant industry protection, the second best policy may actually be to maintain import tariffs or export subsidies forever, even after the industrialized equilibrium is reached. Consumers, who maximize a concave utility function, prefer to spread consumption losses caused by the tariff over the longest possible time. As pre-announced tariffs that will be in effect after the steady state is reached will also have an effect on pre-steady state production decisions, then it is

¹ Bardhan (1971) studies the optimal subsidy plan in a dynamic setting, when there are externalities in production. This paper is different in four main aspects: (i) Bardhan models dynamic externalities (cumulative volume of output enters the production function), whereas in this paper they are static; (ii) this paper studies optimal *trade policy* to deal with the externality problem, which results in a second best equilibrium; (iii) it studies the optimal tariff plan under alternative assumptions about the ability of the government to precommit to future tariffs; and (iv) it includes financial markets, so that the balance of payments does not need to be in equilibrium at all times.

possible extend the distortion beyond that time, i.e., have lower tariffs at all times, instead of higher tariffs before industrialization, and zero tariffs thereafter.

In the other extreme case, when the government's announcement has no credibility at all, the only time consistent policy is a zero tariff in all periods.² The benefit of the tariff, which is its effect on the production decision, happens before the tariff is actually levied, because producers decide how much to invest in the industrial sector based on expected future earnings. The consumption distortion the tariff causes, however, is contemporaneous to it. Hence, a positive tariff is not time consistent.

Only in the intermediate case, when the government can commit to its future economic policy for a limited number of periods, does a sort of "infant industry protection" turn out to be the second best policy.

The results of this paper can also be applied to a more general situation when the government could use subsidies, but not lump-sum transfers to pay for them. If the government could only use income tax, for instance, it would still face the same trade off: introduce subsidies to deal with the externality problem and bear the distorting taxes, or no subsidies and, therefore, no distorting taxes.

²Karp and Paul (1994) reaches, independently, this same conclusion that with no credibility the best time consistent policy is zero tariffs in all periods in a situation similar to the one presented in this model. There are two main differences from that model. The first one is that there the source of externality is in the moving cost - for some reason agents consider average instead of marginal cost when deciding whether to move. The second difference is that the model does not include financial markets, so that consumers cannot reallocate their income intertemporally.

Section 2 introduces the model. In subsection 2.1 the production side of the economy is described, and subsection 2.3 presents the consumption side of the economy. In section 3 the optimal tariff schedule is analyzed in full commitment, no commitment, and limited commitment environments. Section 4 presents the conclusions.

2. The model

A country that is small in the international goods and financial markets is considered. The prices of goods are exogenous, and all economic agents may borrow or lend freely at the exogenous interest rate r . Each individual in this economy is a producer and a consumer. Her decision problem is analyzed in two steps: as a producer she decides how to allocate her labor endowment between sectors, and then, as a consumer, she chooses how much of each good to consume each period. For simplicity, all economic agents are assumed identical, i.e., they have the same per period labor endowment \bar{L} , and they face the same decision problems. The production side of the economy will be studied first.

2.1. Production

The basic framework of the production side of the economy is based on Krugman (1991). There is an economy with two sectors, agriculture and industry. The only factor used in the production of both goods is labor. One unit of the agricultural output A requires one unit of labor. The industrial sector exhibits increasing returns to scale,

external to the firm.³ For simplicity, the externality function is presented as being linear in total labor allocated to the sector. An individual who allocates L units of her labor to the industrial sector will produce output in that sector equal to N , where:

$$N = (\alpha + \beta L^*)L \quad (1)$$

where α and β are constants, and L^* aggregated labor in manufacturing. Note that, because workers are identical, $L^* = nL$, where n is the total number of individuals.

Prices are normalized so that the prices of agricultural and industrial goods are, respectively, $p_A = 1$ and $p_N = p$. It is straightforward to see that if inequalities (2) hold (and it will be assumed here that they do hold) there will be multiple long-run equilibria in this economy.

$$p\alpha < 1 \text{ and } p(\alpha + \beta \bar{L}^*) > 1, \text{ for } \bar{L}^* = n\bar{L} \quad (2)$$

When workers allocate all their time to the industrialized sector, the value of the marginal product of labor in that sector will be larger than that in the agricultural sector, and workers will not want to change their allocation. An analogous situation holds when everyone works only in the agricultural sector. A third possibility arises when labor allocation is such that $p(\alpha + n\beta L) = 1$. This will also be an equilibrium, but an unstable one: any variation in the labor allocation will make the economy go to one of the other two equilibria. If there were no costs of adjustment to labor, there would be another set of equilibria where the economy alternates among periods in each of the three equilibria. The only constraint is that the labor must move at the same time to the same equilibrium.

³ Helpman and Krugman (1990) and Wong (1997) document the importance of economies of scale.

Dynamics are added to the system by introducing some cost for the labor to move from one sector to the other. Labor will not reallocate instantaneously, but will follow some law of motion, governed by the adjustment cost combined with the externality. Following Krugman (1991), the cost is taken to be quadratic in the per period adjustment size:⁴

$$\frac{(L_{\tau+1} - L_{\tau})^2}{2\gamma}, \quad (3)$$

where γ is a constant and L_{τ} is the amount of labor per worker allocated to industry at time τ .

Each worker chooses how to allocate her per period labor endowment between the two sectors maximizing the present value of output:

$$\max \sum_{\tau=0}^{\infty} \left(\frac{1}{1+r} \right)^{\tau} \left[p(\alpha + \beta L_{\tau}^*) L_{\tau} + (\bar{L} - L_{\tau}) - (u_{\tau})^2 / 2\gamma \right], \quad (4)$$

$$\text{subject to } u_{\tau} = L_{\tau+1} - L_{\tau} \quad (4.1)$$

$$0 \leq u_{\tau} + L_{\tau} \leq \bar{L}, \quad (4.2)$$

⁴Mussa (1978) justifies this form of adjustment cost by introducing a moving industry which requires resources to move labor from one sector to the other and presents decreasing returns to scale. Here, the movement cost is motivated by "organizational costs". When changing her labor allocation between the two sectors, the worker needs to reorganize the tasks she performs in each sector, in order to adjust them to the new time constraints. There could be introduced a planning industry which requires resources to reorganize the workers' schedule and presents decreasing returns to scale. Alternatively, it could be assumed that individuals allocate all their per-period labor to only one sector, and some could choose to migrate between sectors. That, however, would introduce an extra source of externality to the model. (See footnote 4.)

where u_τ is the control variable, and $\bar{L} - L_\tau$ represents the amount of labor the worker allocates in agriculture.

When making their labor allocation decision, workers do not perceive the effect of their decision on the productivity of labor in the industrial sector, which, in turn, will affect the income of every worker. Appendix A presents the solution to the maximization problem above. Let ξ_τ denote the Lagrange multipliers for constraints (4.2). They will assume the value zero when the constraints are not binding, and some positive value otherwise. To solve the problem, a future value co-state variable, λ_τ , is introduced. The expression $\frac{\lambda_\tau}{(1+r)^{\tau-1}}$ represents the value today of one more unit of labor in the industrial sector at time τ , and corresponds to the multipliers for constraints (4.1). Along the optimal path, the change in the co-state variable is equal to the difference between its rate of return and the private value of one more unit of labor in the industrialized sector:

$$\lambda_{\tau+1} - \lambda_\tau = r\lambda_\tau - d_\tau, \quad (5)$$

where $d_\tau = p(\alpha + n\beta L_\tau) - 1$ is the difference in labor productivity between the two sectors as perceived by the worker, i.e., without taking into account the externality.

Until an equilibrium is reached, labor will move at any time τ proportionally to the value of the co-state variable:

$$L_{\tau+1} - L_\tau = \gamma \lambda_{\tau+1}. \quad (6)$$

Depending on the initial labor allocation and the parameters of the economy, equilibrium conditions given by the two equations of motion above and the transversality

conditions, derived in appendix A, may lead the economy either to the equilibrium with specialization in industry, or the one with specialization in agriculture. Equation (7) presents the terminal condition for the co-state variable if the economy is heading to the industrialized equilibrium, while equation (8) gives the terminal condition if it is heading to the agriculture one.⁵

$$\lambda^N = \frac{p(\alpha + n\beta\bar{L}) - 1}{r} \quad (7)$$

$$\lambda^A = \frac{p\alpha - 1}{r} \quad (8)$$

It is straightforward to see that the labor movement resulting from a free market is different from that chosen by a central planner, because of the externality in production. Workers underestimate the benefit from moving into the industrial sector, because they do not account for the effect of their movement on the productivity of the sector, and therefore on the income of every worker. The central planner's solution to the maximization of income implies labor movement as the same function of the co-state

⁵See appendix A for the derivation of equations (7) and (8). The terminal conditions here are different from those derived in Benabou and Fukao (1993) for Krugman's model. The difference is due to their interpretation of the Krugman model as having each worker producing either the industrial or the agricultural good (not both, as in the present model). The moving cost, then, depends on the total number of workers changing sectors. When choosing whether to migrate or not, the worker does not take into account the effect of her decision on the adjustment cost, so that it becomes another source of externality in the model.

variable as in equation (6), but the movement of the co-state variable is given by equation (9) instead of (5):

$$\lambda_{\tau+1} - \lambda_{\tau} = r\lambda_{\tau} - d_{\tau}^P, \quad (9)$$

where $d_{\tau}^P = p(\alpha + 2n\beta L_{\tau}) - 1$ is the difference between the value of the marginal product of labor in the two sectors as perceived by the central planner. Note that the value for the central planner of moving one more unit of labor into industry at each period (d_{τ}^P) is greater than the value for the worker (d_{τ}).

The central planner's terminal conditions for the co-state variable if the economy is heading to the industrialized or agricultural equilibria are given by equation (10).

$$\begin{aligned} \lambda^N &= \frac{p(\alpha + 2n\beta \bar{L}) - 1}{r} \\ \lambda^A &= \frac{p\alpha - 1}{r} \end{aligned} \quad (10)$$

To help the comparison between the free market and central planner solutions to the problem, a graphic interpretation of the dynamics is presented.

Figure 1 shows the shape of the paths leading to the long run equilibria, derived from the system of difference equations defined by the first order conditions from the workers' decision (equations (5) and (6)), and the terminal conditions (equations (7) and (8)). The variables will follow discrete points along the paths. The continuous path is the limit as the time interval between periods (which is taken as being '1') goes to zero.

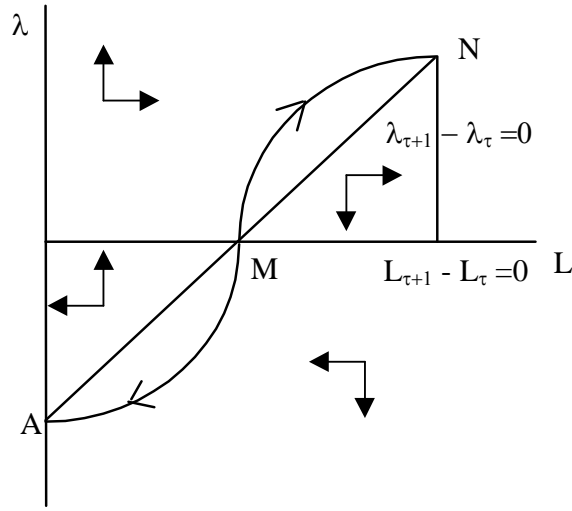


Figure 1

The two roots of the system defined by equations (5) and (6),

$$\mu_1 = 1 + \frac{r + \sqrt{r^2 - 4(1+r)\gamma\beta Np}}{2} \text{ and } \mu_2 = 1 + \frac{r - \sqrt{r^2 - 4(1+r)\gamma\beta Np}}{2},$$

are greater than 1; therefore the system will diverge from its only singular point, represented by M in the figures. Point M is a source, that is, all the characteristics of the system originate from that point, moving away from it. The labor boundary conditions create two stable equilibria: A and N. At N, the value of co-state variable is positive (from (7)), that is, the value today of one additional unit in the industrialized sector at the time the economy is specialized in its production is positive. Hence, the workers would be willing to allocate more of their labor endowment to that sector. As they have no more labor available, they do not move from that point. Point A is analogous, with a negative value of the co-state variable. These points are denoted "corner" solutions.

For an initial labor allocation, the value of the co-state variable λ_τ will determine the path the economy will follow. In other words, the value of one more unit of labor in the industrial sector (λ_τ) will determine how much labor moves, and, together with the current amount of labor in this sector, it will determine how much its own value changes. The shape of the path in the figure represents the dynamics when the roots of the system are real.⁶

For initial labor allocations to the right of point M, the productivity in industry is high enough so that λ_1 assumes a positive value, and the economy heads to the industrialized equilibrium. The opposite is true for initial labor allocations to the left of point M.

Now the central planner's solution is compared to the free market dynamics. The $L_{\tau+1} - L_\tau = 0$ schedule is the same in the two cases, since it must coincide with $\lambda = 0$ in both cases. But the value of one more unit of labor in the industrial sector is greater for the central planner for each allocation of labor. Therefore the $\lambda_{\tau+1} - \lambda_\tau = 0$ line for the central planner is located up and to the left compared to the market one.

⁶Krugman (1991) presents an interesting interpretation of the dynamics when the roots of the system are imaginary in a continuous time analog of the present model.

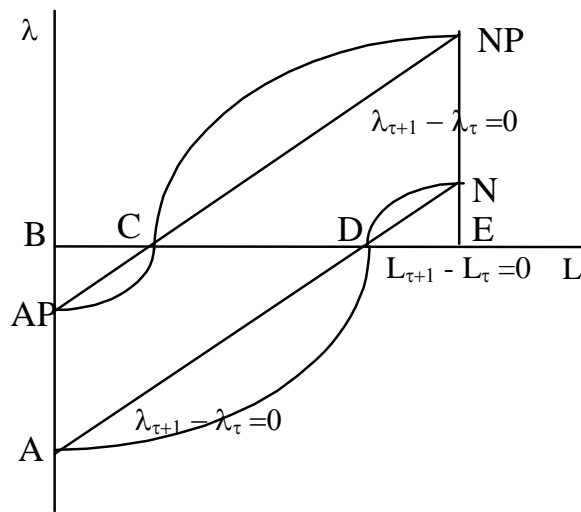


Figure 2

In figure 2, jP and j , for $j=A, N$, represent the equilibrium points where the economy specializes in sector j resulting from the central planner and free market dynamics, respectively.

The marginal product of labor in industry for the central planner includes the effect of labor movement on the productivity of every worker, which is not taken into account by the individual worker. Therefore for each labor allocation the value of the marginal product of labor in industry is higher for the central planner than for the workers. If the initial labor allocation lay between points C and D in figure 2, then this difference would be decisive to determine which long run equilibrium was to be reached. The central planner would lead the economy to the industrialized equilibrium, whereas the free market would specialize in agriculture.

For initial labor allocation between points B and C , there is so little labor in industry that even the central planner considers not worth moving into that sector. Both

the central planner and the free market would go to the equilibrium with specialization in agriculture, however the central planner would do so at a slower rate.

Finally, if the initial labor allocation were to the right of point D, central planner and the free market would head to the industrialized equilibrium. The central planner would move at a faster rate due to his higher valuation of the marginal product of labor in industry for each labor allocation.

The first best economic policy to implement the central planner's solution is to introduce a production subsidy financed by lump-sum transfers from the consumers. The value of the subsidy at each point in time should be the one that makes the producer follow the central planner's equilibrium path, given the initial labor allocation. The subsidy would make the workers' perception of the value of shifting labor between sectors equal to that of the central planner, and it would have no other effect in the rest of the economy.

This paper considers the situation in which trade policies are the only instruments available to the policy makers. An import tariff and export subsidy would mimic the effect of the production subsidy from the producer's point of view, but it would also affect the consumption decision, causing utility diminishing distortions. Therefore, to derive the optimal trade policy, it is necessary to study its effects on the welfare of the economy as a whole, not only on its production side.

2.3. Consumption

The government will set some import tariff/export subsidy schedule, taking as given the production and consumption decisions of the individuals. Workers and

consumers face the price $p(1 + v_\tau)$ for the industrial good, where v_τ is the value of the ad valorem import tariff/export subsidy imposed at time τ . Workers choose how to allocate their labor between the two sectors, and with the income they receive they decide how much to consume of each good. Each worker is taken to be identical, as they face the same decision problem, and therefore make the same decisions. They are taken to be identical consumers as well, by assuming they have the same utility function. Due to the symmetry of the problem, maximizing welfare is equivalent to maximizing utility for the representative consumer, which is represented by:

$$U^i = U = \sum_{\tau=0}^{\infty} \frac{1}{(1+\theta)^\tau} [a \log C_\tau^N + b \log C_\tau^A] \quad (14)$$

where C_τ^j is the consumption of good j at time τ , and θ is the discount rate.

The consumer has free access to the international financial market at the world interest rate r . In each period the increment to her asset holdings equals the interest on the assets held at the beginning of the period, plus the difference between her total income and her total expenditures, plus any transfer received from, or given to, the government. By assumption, no assets are held at the start of the initial period. Hence, consumers maximize utility subject to the following constraints:

$$\begin{aligned} f_{\tau+1} - f_\tau &= r f_\tau + I_\tau^p - p(1 + v_\tau) C_\tau^N - C_\tau^A + R_\tau, \text{ and} \\ f_0 &= 0, \end{aligned} \quad (15)$$

where total income received from production is $I_\tau^p = p(1 + v_\tau) N_\tau + A_\tau - \frac{\gamma(\lambda_{\tau+1})^2}{2}$, f_τ is the amount of assets the consumer holds at the start of period τ , and R_τ is the lump sum

transfer from the government at each period. The government's balanced budget requires

$$\text{that } \sum_{i=0}^{\infty} (1+r)^{-i} [pv_i(C_i^N - N_i) - R_i] = 0.$$

The following Hamiltonian is used to solve the problem:

$$H = (a \log C_\tau^N + b \log C_\tau^A)(1+\theta)^{-\tau} + \bar{y}_{\tau+1} [rf_\tau + I_\tau^p - p(1+v_\tau)C_\tau^N - C_\tau^A + R_\tau] \quad (16)$$

where $\bar{y}_{\tau+1}$ is the co-state variable, which is interpreted as the value today of having one more unit of asset at time $\tau + 1$.

The first order conditions for the maximization above are:

$$\frac{a}{C_\tau^N} - y_{\tau+1}p(1+v_\tau) = 0, \quad (17.a)$$

$$\frac{b}{C_\tau^A} - y_{\tau+1} = 0, \quad (17.b)$$

$$y_{\tau+1} - y_\tau = \left(\frac{\theta - r}{1+r} \right) y_\tau, \text{ and} \quad (17.c)$$

$$f_{\tau+1} - f_\tau = rf_\tau + I_\tau^p - p(1+v_\tau)C_\tau^N - C_\tau^A + R_\tau, \quad (17.d)$$

where $y_{\tau+1} \equiv \bar{y}_{\tau+1}(1+\theta)^\tau$ is the future value co-state variable.

Equation (17.c) shows that the value of y_τ will change over time if, and only if, the interest rate is different from the rate of time preference. As having this variable changing over time would not add any insight to the analysis, $r = \theta$ is assumed from now on. This assumption implies perfect consumption smoothing in both goods.

Furthermore, the transversality condition states that as time goes to infinity either the present value of one more unit of future asset (the co-state variable) goes to zero, or no assets are being held:

$$\lim_{\tau \rightarrow \infty} \frac{y}{(1 + \theta)^\tau} f_\tau = 0. \quad (18)$$

Now the model is complete: the decision problem of consumers and producers is solved, and the best tariff schedule can be derived.

3. Optimal Tariff

Three different environments will be considered: the first is when the government is able to commit to its tariff schedule for the future, the second is the opposite case, when the government cannot commit to any future tariff, and, finally, the third situation is when the government has limited commitment, i.e., it can commit only for a certain number of periods. In each case the indirect utility function that the policy maker maximizes when setting tariffs, which is the indirect utility function for the representative individual, is derived.

3.1. Full Commitment

When the government is able to fully commit to its tariff plan, it will decide in the initial period on a tariff schedule for the whole future, maximizing the indirect utility function over that period of time, i.e., forever. First this indirect utility function will be derived.

Equation (19) is derived from equation (17.d) and the fact that no assets are held at the initial time:

$$f_\tau = \sum_{i=0}^{\tau} (1+r)^{\tau-i} [I_i^p - p(1+v_i)C_i^N - C_i^A + R_i]. \quad (19)$$

Using the government's budget constraint and substituting equations (17.a) and (17.b) into equation (19) yields:

$$f_\tau = \sum_{i=0}^{\tau} (1+r)^{\tau-i} \left[I_i^p - \frac{a+b}{y} + R_i \right]. \quad (20)$$

The value of the co-state variable is obtained by substituting f_τ from equation (20) into the transversality condition, and using the government's budget constraint. The result is:

$$y = \frac{S_0}{PVI_0}, \quad (21)$$

where $S_0 = \sum_{i=0}^{\infty} (1+r)^{-i} \left[\frac{a}{1+v_i} + b \right]$ can be interpreted as a measure of the present value of the consumption distortion caused by the tariff, $PVI_0 = \sum_{i=0}^{\infty} (1+r)^{-i} I_i$ is the present value of income, and $I_i = pN_i + A_i - \gamma\lambda_{i+1}^2/2$ is the income from production without the effect of the tariff.

The indirect utility function in equation (22) is derived by substituting the value of y into equations (17.a) and (17.b) to calculate the levels of consumption, and then substituting these into the utility function:

$$V_0(L_\tau, v_\tau) = K + \frac{(1+\theta)(a+b)}{\theta} (\log PVI_0 - \log S_0) - a \sum_{i=0}^{\infty} (1+\theta)^{-i} \log(1+v_i), \quad (22)$$

where K is a constant term.

Note that equation (22) represents the indirect utility function for the whole stream of consumption from period zero to infinity.

Before proceeding with the maximization of equation (22), the effect of tariffs on the present value of income needs to be studied. The present value of income is a function of the labor allocation L and the value of the co-state variable λ in each period. An expression for each period labor allocation will be derived.

As shown in appendix A, the value of moving one more unit into industry in period τ is:

$$\lambda_{\tau+1} = \sum_{i=\tau+1}^{\infty} \frac{d_i^v}{(1+r)^{i-\tau}}, \quad (23)$$

where $d_i^v = p(1+v_i)(\alpha + n\beta L_i) - 1$ is the difference in marginal labor productivity in the two sectors as perceived by the worker, incorporating the effect of the tariff.

Using the result above, each period's labor movement (from equation (6)) may be rewritten as a function of future tariffs and labor allocations.:

$$L_{\tau+1} - L_\tau = \gamma \sum_{i=\tau+1}^{\infty} \frac{d_i^v}{(1+r)^{i-\tau}}, \quad (24)$$

which yields:

$$L_\tau = L_0 + \gamma \sum_{j=1}^{\tau} \left[\sum_{i=j}^{\infty} \frac{d_i^v}{(1+r)^{i-j+1}} \right] \text{ for } 1 \leq \tau < T. \quad (25)$$

From (25), the path of future tariffs affects labor allocation today. At time T an equilibrium is reached, such that:

$$L_{T+s} = L_0 + \gamma \sum_{j=1}^T \left[\sum_{i=j}^{\infty} \frac{d_i^v}{(1+r)^{i-j+1}} \right] = \begin{cases} \bar{L} & \text{if the upper boundary is reached} \\ 0 & \text{if the lower boundary is reached} \end{cases} \quad (26)$$

for $s \geq 0$

The system of equations represented in equation (25) has a unique solution for a set of tariff schedules. The government will choose the one that maximizes welfare.

Given the values of the initial and final labor allocations, L_0 and L_T , and the value of T , the system of equations represented in equation (25) can be solved to get the entire labor allocation stream as a function of the tariffs schedule. The indirect utility function can be written as a function of the tariff schedule only.⁷

$$V_0(L_\tau, v_\tau) = V(L_\tau, \{v_i\}_0^\infty, v_\tau) \quad (27)$$

To set the conditions for the welfare maximizing tariff plan, all we need is the tariff schedule that makes the derivative of the indirect utility function equal zero, i.e., that satisfies:

$$\frac{(1+\theta)(a+b)}{\theta PVI_0^*} \frac{\partial PVI_0}{\partial v_\tau} = \left[1 - \frac{(1+\theta)(a+b)}{\theta S_0^*(1+v_\tau^*)} \right] \frac{a}{(1+v_\tau^*)(1+r)^\tau} \forall \tau \geq 0 \quad (28)$$

where v_τ^* is the optimal tariff at time τ , and PVI_0^* and S_0^* represent the value for those variables when the optimal tariff plan is followed.

⁷ Note, again, that equation (27) represents the indirect utility function for the stream of consumption from period zero to infinity.

The left hand side of equation (28) represents the effect of the tariff on the present value of income. Producers' decisions determine the present value of income, but those decisions are not optimal from the central planner's point of view because the producers do not consider the effect of their decision on the productivity of industry (review the difference between equations (5) and (9)). A positive value for the tariff will bring the producers' decisions closer to the central planner's solution, thereby increasing the present value of income.

The right hand side represents the loss in utility brought about by the tariff due to the consumption distortion. It depends not only on the current tariff, but also on the whole stream of tariffs through the term S_0^* . This means that if, for some reason, only the tariff at some particular time t affected the PVI_0 , (i.e., if the l.h.s. were different from zero only in period t), nevertheless the optimal tariff would be different from zero at all times. Due to the concavity of the utility function, consumers would prefer to spread the consumption distortion over the time, instead of having it concentrated in only one period.

The solution to the problem as stated above yields the utility maximizing tariff schedule given that the labor frontier is to be reached in T periods. The problem can then be solved for all possible values of T , and the welfare maximizing tariff schedule is the one with a value for T that achieves the highest utility. Proposition 1 states two main characteristics of the optimal tariff schedule.

Proposition 1: Under full commitment $v_\tau^* > 0 \forall \tau \geq 0$, and $v_{T+s}^* = v_T^* \forall s \geq 0$, where T is the time at which the labor boundary is reached.

Proof: First we will show that $\frac{\partial PVI_0}{\partial v_\tau} > 0 \forall \tau > 0$. The only difference between the central planner's and the market's solution to the maximization of the present value of income is their perception of the value of the difference in labor productivity between the two sectors. That value in the central planner's solution, d_τ^p in equation (9), is greater than the one perceived by the worker, d_τ in equation (5). A positive tariff no greater than $\frac{n\beta L_\tau}{\alpha + n\beta L_\tau}$ approaches the difference perceived by the worker under tariff, d_τ^v in equation (23), to the one perceived by the central planner, and therefore resulting in a greater present value of income. Given that $\frac{\partial PVI_0}{\partial v_\tau} > 0 \forall \tau > 0$, equation (28) is satisfied if and only if $v_\tau^* > 0 \forall \tau > 0$.

For $\tau = 0$, $\frac{\partial PVI_0}{\partial v_0} = 0$, and equation (28) is satisfied if and only if $1 + v_0 = \frac{(1+\theta)(a+b)}{\theta S_0^*}$. The expression $\frac{(1+\theta)(a+b)}{\theta S_0^*}$ would equal 1 if the tariff were zero at all periods, and would be greater than 1 for positive tariffs in any period. Given that $v_\tau^* > 0 \forall \tau > 0$, then $S_0^* < \frac{(1+\theta)(a+b)}{\theta}$, and therefore it must be the case that $v_0 > 0$.

As for the second statement of proposition 1, Appendix B proves that:

$$\frac{\partial PVI_0}{\partial v_{T+s}} = (1+r)^{-s} \frac{\partial PVI_0}{\partial v_T} \quad \forall s \geq 0. \quad (29)$$

Therefore, for the optimal tariff after the labor boundary is reached, equation (25) can be rewritten as :

$$\frac{(1+\theta)(a+b)}{\theta PVI_0^*} \frac{\partial PVI_0}{\partial v_T} = \left[1 - \frac{(1+\theta)(a+b)}{\theta S_0^*(1+v_{T+s}^*)} \right] \frac{a}{(1+v_{T+s}^*)(1+r)^T} \forall s \geq 0. \quad (30)$$

For the r.h.s. to be positive it is necessary that $1 - \frac{(1+\theta)(a+b)}{\theta S_0^*(1+v_{T+s}^*)} > 0$, which is equivalent to $1 + v_{T+s} > \frac{(1+\theta)(a+b)}{\theta S_0^*}$, and the condition is satisfied if and only if $v_{T+s} > 0$.

Equation (30) must hold for all v_{T+s} , and the other variables in the equation are constant $\forall s \geq 0$, except for v_{T+s} . Therefore, the tariff must be constant over this period of time. Moreover, the left hand side of the equation is constant and strictly greater than zero, so that the tariff must be positive for the right hand side to be also positive.

Q.E.D.

Proposition 1 states that the optimal import tariff/export subsidy will be always positive, moreover it will be constant after the labor boundary is reached.⁸

Tariffs should be positive forever due to the convexity of the deadweight consumption loss caused by it, so that the best policy is to spread the consumption losses over the longest possible time. In the present model, it is possible to have lower tariffs over a long horizon, instead of higher tariffs during the transition, and zero tariffs thereafter, because all future tariffs affect production decisions today .

Another way to explain this result is the following. A small tariff after the steady state is reached at some finite time T would have a positive first order welfare effect, due

⁸This result refers to the tariff schedule that maximizes welfare. If one seeks for a tariff schedule that just ensures industrialization, many others would do the job, including an infant industry protection.

to its first order effect on labor migration before the steady state, and only a second order consumption cost. Therefore, zero tariffs after the steady state cannot be optimal.

The result that the optimal tariff will be constant after the boundary is reached relies on the fact that tariffs from that moment on will not affect the present value of income, PVI , by affecting present or future labor allocations, because labor will remain static at the frontier (i.e., $\frac{\partial \mathcal{L}_{T+\tau}}{\partial v_{T+s}} = 0 \forall \tau \geq 0$ and $\forall s \geq 0$). They affect the present value of income only through labor allocation decisions prior to reaching the boundary. Therefore the effect of tariffs in different periods after time t^c will be equal except for the different discount applied to each one, i.e., the result in equation (29). The effect of tariffs levied after time T on the consumption distortion, on the other hand, will also be equal except for the discount rate applied to each period. As the optimal tariff is the one that equalizes the marginal gain and the marginal distortion caused by it, it will be constant after the boundary is reached.

If this were not a "corner" solution, this result would not hold, i.e., tariffs at the equilibrium point would be able to affect the position of the economy so that $\frac{\partial \mathcal{L}_{T+\tau}}{\partial v_{T+s}} \neq 0 \forall \tau \geq 0$ and $\forall s \geq 0$. Tariffs would still be positive at equilibrium, but they would not necessarily be constant.

3.2. No Commitment

When the government is not able to commit to a tariff schedule for the whole future, it chooses, at each moment in time, the tariff that will maximize welfare from that moment on. The optimal tariff plan under full commitment is not time-consistent.

Equation (23) shows that at any period τ the decision on how much labor to move into industry for the next period depends only on *future* tariffs. On the other hand, the consumption distortion created by the tariff is contemporaneous to it. Therefore, the government will have an incentive to announce the tariff consistent with the desired rate of labor adjustment, but by the time of its implementation the labor movement would already have taken place, and the best thing to do would be to set zero tariffs. Proposition 2 presents the time consistent solution.

Proposition 2: With no commitment $v_\tau^* = 0 \forall \tau \geq 0$.

Proof: First the optimal tariffs schedule after the labor boundary is reached will be derived. The indirect utility function the government will be facing at time T is:⁹

$$V_T(L_\tau, v_\tau) = \bar{K} - \frac{(1+\theta)(a+b)}{\theta} \log S_T - a \sum_{i=T}^{\infty} (1+\theta)^{T-i} \log(1+v_i), \quad (31)$$

where
$$S_T = \sum_{i=0}^{\infty} (1+r)^{-i} \left[\frac{a}{(1+v_{T+i})} + b \right].$$

The only variable in this new indirect utility function is the tariff. There is no labor movement after the boundary is reached, and therefore tariff changes will not affect the present value of income from that moment on. The first order conditions for maximization are:

$$\left[1 - \frac{(1+\theta)(a+b)}{\theta S_T^* (1+v_{T+s}^*)} \right] \frac{a}{(1+v_{T+s}^*)(1+r)^s} = 0 \quad \forall s \geq 0. \quad (32)$$

⁹See the derivation of this equation in appendix C.

The first term of the product above must equal zero for the condition to be satisfied. The value of the tariff is the only variable in this term, which means that the value of the tariff that satisfies the condition is the same over time, i.e., $v_{T+s} = v_T \quad \forall s \geq 0$.

Hence, using the assumption that $\theta = r$, the tariff must satisfy $\frac{(1+\theta)bv_T^*}{\theta(1+v_T^*)} = 0$, which will

be true if and only if $v_T^* = 0$. Thus, the tariff will be set equal to zero when the boundary of labor supply is reached, regardless the previous tariff schedule.

Taking one step back, the optimal tariff to be set one period before the boundary is reached is derived. At time $T-1$ workers decide how much labor to move into industry based on the value of λ_{T-1} (see equation (6)), which in turn will be given by equation (23), and only tariffs after time $T-1$ enter this equation. It is then clear that the value of the tariff at time $T-1$ will not affect labor allocation in that or in any future period, and hence will not affect income either. Therefore, the condition for maximization of welfare from period's $T-1$ perspective will be analogous to equation (32):

$$\left[1 - \frac{(1+\theta)(a+b)}{\theta S_{T-1}^* (1+v_{T-1}^*)} \right] \frac{a}{(1+v_{T-1}^*)} = 0 \quad (33)$$

where $S_{T-1}^* = \frac{a}{1+v_{T-1}^*} + \frac{a}{r} + b\left(\frac{1+r}{r}\right)$, given the result above that $v_{T+s} = 0 \quad \forall s \geq 0$.

Again, equation (31) will be satisfied if, and only if, $v_{T-1} = 0$. Going backwards in time the same situation arises: at each period the best trade policy from that period's perspective is a zero tariff.

Q.E.D.

The optimal tariff plan in this no commitment case yields a third best outcome. The government lacks the policy instrument that could make possible the achievement of the first best outcome: subsidies. The use of "surprise", or diverging from the pre-announced policy, works as an additional instrument to try to improve on the second best outcome. However, the agents predict this temptation, and act accordingly: they make their decisions expecting zero tariffs in the future. The only time consistent plan is the one with zero tariffs forever.

3.3. Limited Commitment

Instead of the two extreme cases discussed above, it may be more realistic to think of the government as having the ability to commit to a policy for a limited period of time. In a democracy, the government changes periodically. It can try to create rules that make it difficult for the next government to change its policies, but it cannot fully commit to an economic policy after its term in office is over.

I thus ask which would be the best time consistent trade policy under the current model if the government can commit to its policy for some number of periods h , where $h > 0$. The dynamics work as follows: at the start of the initial period the government sets a tariffs schedule for h periods, before the end of the h^{th} period, but after labor allocation decisions are made in period $h - 1$, it sets the tariffs for the next h periods, and so on. Proposition 3 summarizes the optimal tariff schedule for this situation.

Proposition 3: If the government is able to commit to its policy plan for h periods, so that each h periods it announces the policy for the next h periods, then tariffs will be strictly positive for every period until the last period of the h period interval

that includes the time the economy reaches the steady state, and tariffs will be zero thereafter. Moreover, the longer the commitment interval h , the higher the optimal tariff before the interval that includes the time when the steady state is reached. Formally, $v_\tau^* > 0$ for $\tau < kh$, and $v_\tau^* = 0$ for $\tau \geq kh$, where k is the highest positive integer such that $k - 1 < t/h$. Moreover, $v_\tau^* > v'_{\tau^*}$ where v_τ^* is the optimal tariff if the commitment interval is h periods, and v'_{τ^*} if the interval is h' periods, for $h' < h$, $\tau < \min\{kh, k'h'\}$, and $\max\{kh, k'h'\} < T$.

Proof: To prove proposition 3 the same strategy as in the "no commitment" case will be used: work backwards in time in the model. Given that the long run equilibrium is reached at time T , the indirect utility function which the government will maximize every h periods to choose the tariff schedule for the h -period interval is:¹⁰

$$V_t(L_\tau, v_\tau) = \bar{K} + \frac{(1+\theta)(a+b)}{\theta} (\log PVI_t - \log S_t) - a \sum_{i=t}^{t+h} (1+\theta)^{-i} \log(1+v_i) \quad (34)$$

where $PVI_t = \sum_{i=t}^T (1+r)^{-(i-t)} I_i + \frac{I^E}{r(1+r)^{T-t}}$, with I^E the (constant) value of the per

period income at one of the long run equilibria, $S_t = \sum_{i=t}^{\infty} (1+r)^{-(i-t)} \left(\frac{a}{1+v_i} + b \right)$, and \bar{K} is

a constant.

The first order conditions for maximization are:

¹⁰Equation (34) is derived similarly to equation (31). At some time t future utility is maximized, taken as given stocks accumulated until that time, and, here, also given the tariffs after time $t+h$.

$$\begin{aligned} \frac{(1+\theta)(a+b)}{\theta PVI_t^*} \frac{\partial PVI_t}{\partial v_\tau} &= \\ &= \left[1 - \frac{(1+\theta)(a+b)}{\theta S_t (1+v_\tau^*)} \right] \frac{a}{(1+v_\tau^*)(1+r)^{\tau-t}} \quad \forall \tau, t \leq \tau < t+h \end{aligned} \quad (35)$$

For $k \geq T/h$ the l.h.s. of equation (35) is equal to zero. Therefore, using the same argument as in the previous section, $v_\tau^* = 0$ for $\tau \geq kh$, $k \geq T/h$. For $k-1 < T/h$ the l.h.s. of equation (35) will be positive, hence positive tariffs will be required for the equation to be satisfied. Finally, the larger h , more elements will be included in the l.h.s. sum, therefore the larger will be the tariff each period over the interval to satisfy equation (35).

Q.E.D.

An important result in this proposition is that the greater the number of periods over which a government can make commitments, the greater the tariff in each period should be. The limiting cases are the ones described in propositions 1 and 2: with no commitment tariffs are always zero, and with full commitment tariffs are positive forever.

The welfare resulting from the limited commitment time consistent policy is higher than the no commitment one.¹¹ Because of the possibility of some commitment to future policy, the government can use, at least partially, trade policy to diminish the loss from the externality problem. But the result should be still worse compared to the full commitment case, when trade policy can be explored fully to deal with the externality.

¹¹ The fact that the government chooses positive tariffs in the limited commitment case, when zero tariffs is also a possible choice, reveals that welfare is not lower under limited commitment than under no commitment.

4. Conclusion

This paper develops a model in which there are two sectors: agriculture and industry. The industrial sector presents positive externalities in production, and there are adjustment costs to changing production from one sector to the other. The model shows that, although the equilibrium with specialization in the industrial good is strictly better than the equilibrium with specialization in agriculture, the initial labor allocation may have so little labor in industry that the present value of income is maximized with the economy following the path to the agricultural equilibrium. The externality in the production of the industrial good distorts the incentives for workers to shift labor into that sector. There is a labor allocation range over which the central planner would take the economy to the industrial equilibrium, while the market on its own would head toward agriculture. Thus, economic policy could not only make workers shift their labor at a rate that raises the present value of income, but could also lead the economy to a different equilibrium than the one it would go to on its own. The paper analyses how trade policy should be used in this setting.

The most striking result of this paper is the one presented in proposition 1, which states that if the government can make credible, indefinite commitments, the best trade policy is to keep protection forever, even after the long run equilibrium is reached, and even if the equilibrium reached is the agricultural one! The intuition behind this result is the following. Although trade policy can improve incentives on the production side of the economy, it also causes welfare diminishing distortions on the consumption side. The optimal import tariff/export subsidy finds the best balance between the two effects. As

our infinitely lived agents have strictly concave utility functions, they would like to smooth their consumption. Therefore, instead of higher tariffs during the transition to the steady state and zero tariffs after it is reached, the best policy here would be lower tariffs during the transition and constant positive tariffs after equilibrium is reached. In this way the proper incentives to the workers would be provided, but the “consumption cost” would be spread over time.

Proposition 2 highlights the importance of precommitment. It shows that if the government has no commitment capability the tariff will always be zero, i.e., the government will not be able to use trade policy to achieve a higher income level.

The result from proposition 3 seems to be more realistic. It states that when the government has limited commitment to its future policy tariffs will be lower than those with full commitment, and they will eventually be zero, after equilibrium is reached. The result is still worse for our economy, compared to the case the government has full credibility.

An interesting development of these results is that institutions that give credibility to a government's long term policy plan could be welfare-improving. Take, for instance, protectionist lobbying. Given that it may be a credible guarantor of protection over some period of time, its presence may be welfare-improving if the government does not have much policy credibility. A word of caution is necessary here: if the lobbying is not believed to be effective over the medium term, but is able to raise trade barriers, then the worst result of all arises. Workers will make their labor allocation decisions given that they expect no protection in the future, so that the (unexpected) protection does not improve labor allocation, but still causes consumption distortions.

What about a developing country that inherits trade barriers on industrial goods, and is on its way to the industrialized equilibrium? If the government is to restructure its trade policy, the first best alternative is to choose import tariffs/export subsidies so as to maximize the indirect utility function, as in section 3.1. Given that the government can commit to its policy for some number of periods h , this would imply positive tariffs, even for some time after complete industrialization. However, one has to take into account the effect of changing current trade policy on the government's credibility vis-à-vis the new tariff plan it announces. A result like proposition 2 may arise.

Appendix A

In this appendix the producer's problem is solved, which is the maximization of the present value of income (equation (4)) subject to the constraint on the movement of the state variable (equation (4.1)), and the per-period constraints on the state variables (equation (4.2)). Equation (4.2) yields two possible cases: the lower boundary is binding, in which case the constraint is $\check{G}(L, u) = -u_\tau - L_\tau \leq 0$; or the upper boundary is binding, in which case it is $\hat{G}(L, u) = u_\tau + L_\tau - \bar{L} \leq 0$.

The following Hamiltonian is used to solve the producer's maximization problem:

$$H(L, u, \lambda, \tau) = [d_\tau L_\tau + \bar{L} - u_\tau^2 / 2\gamma](1+r)^{-\tau} + \bar{\lambda}_{\tau+1} u_\tau \quad (\text{A1})$$

where $d_\tau = p(\alpha + n\beta L_\tau) - 1$, and $\bar{\lambda}_{\tau+1}$ is the co-state variable, representing the value today of one more unit of labor in the industrial sector at time τ .

The first order conditions necessary and sufficient¹² for maximization are:

(a) the control variable (u_τ) at each period must be chosen to maximize the function $H(L, u, \lambda, \tau)$, subject to constraint $\check{G}(L, u)$ or $\hat{G}(L, u)$, depending on which one is binding;

(b) the state and co-state variables must change over time according to the following equations:

$$\begin{aligned}\bar{\lambda}_{\tau+1} - \bar{\lambda}_\tau &= -H_L^*(L_\tau, u_\tau, \bar{\lambda}_{\tau+1}, \tau) \\ L_{\tau+1} - L_\tau &= H_\lambda^*(L_\tau, u_\tau, \bar{\lambda}_{\tau+1}, \tau)\end{aligned}\tag{A2}$$

where $H^*(L_\tau, u_\tau, \lambda_{\tau+1}, \tau)$ is the value of the Hamiltonian after u_τ is optimally chosen.

There are two different sets of conditions for part (a), depending on which constraint is binding, i.e., whether the economy will reach the agricultural or industrialized equilibrium in the long run.

If the economy is heading to the agricultural equilibrium, the Lagrangian $\mathfrak{S} = H(L, u, \lambda, \tau) + \check{\xi}_\tau \check{G}(L, u)$ is used, where $\lambda_{\tau+1} \equiv \bar{\lambda}_{\tau+1}(1+r)^\tau$, and the following condition from part (a) is derived:

¹²The following conditions are sufficient, as well as necessary, because the Hamiltonian maximized with respect to the control variable is a concave function of the state variable. (See Intrilligator (1971), p.366, fn.

$$\frac{u_\tau}{\gamma} - \lambda_{\tau+1} - (1+r)^\tau \tilde{\xi}_\tau \leq 0, \quad -u_\tau \geq 0 \quad \text{with complementary slackness,} \quad (\text{A3})$$

$$u_\tau + L_\tau \geq 0, \quad \tilde{\xi}_\tau \geq 0 \quad \text{with complementary slackness.}$$

And if the economy is heading to the industrialized equilibrium, the Lagrangian used is $\mathfrak{S} = H(L, u, \lambda, \tau) + \widehat{\xi}_\tau \widehat{G}(L, u)$, and the conditions:

$$-\frac{u_\tau}{\gamma} + \lambda_{\tau+1} - (1+r)^\tau \widehat{\xi}_\tau \leq 0, \quad u_\tau \geq 0 \quad \text{with complementary slackness,} \quad (\text{A4})$$

$$u_\tau + L_\tau - \bar{L} \geq 0, \quad \widehat{\xi}_\tau \geq 0 \quad \text{with complementary slackness.}$$

Finally, the equations for part (b) are:

$$\lambda_{\tau+1} - (1+r)\lambda_\tau = -d_\tau \quad (\text{A5})$$

$$L_{\tau+1} - L_\tau = u_\tau \quad (\text{A6})$$

The transversality conditions also need to be satisfied. These conditions state that as time goes to infinity either the value of moving one more unit of labor into a sector is zero, or there is no labor in that sector. Equation (A7) presents the transversality conditions for the industrialized and agricultural equilibria, respectively.

$$\lim_{\tau \rightarrow \infty} \frac{\lambda_{\tau+1}}{(1+r)^\tau} L_\tau = 0 \quad \text{or} \quad (\text{A7})$$

$$\lim_{\tau \rightarrow \infty} \frac{-\lambda_{\tau+1}}{(1+r)^\tau} (\bar{L} - L_\tau) = 0$$

Equations (A3)-(A7) completely determine the solution to the problem. Using equations (A3) and (A4), labor movement will follow:

$$L_{\tau+1} - L_\tau = \gamma \lambda_{\tau+1}, \quad (\text{A8})$$

when the labor frontier is not binding

When one of the frontiers is reached, the control variable is set to zero due to equation (A6), $u_\tau = 0$, and either $(1+r)^\tau \tilde{\xi}_\tau \geq -\lambda_{\tau+1}$ or $(1+r)^\tau \widehat{\xi}_\tau \geq \lambda_{\tau+1}$, depending on whether the lower or upper boundary is reached, respectively.

From equation (A5), and using the transversality conditions:

$$\lambda_{\tau+1} = \sum_{i=\tau+1}^{\infty} \frac{d_i}{(1+r)^{i-\tau}}. \quad (\text{A9})$$

The value of the co-state variable when the economy is at the long run equilibrium is derived using equation (A9):

$$\lambda^k = \frac{d^k}{r} \quad \text{for } k = N, A \quad (\text{A10})$$

where $d^N = p(\alpha + n\beta\bar{L}) - 1$ and $d^A = p\alpha - 1$.

Appendix B

In this appendix the validity of equation (26) will be proved. The value of equation (B1) will be derived.

$$\frac{\partial PVI}{\partial v_{T+s}} = \sum_{i=1}^{T-1} \frac{\partial PVI}{\partial \mathcal{L}_i} \frac{\partial \mathcal{L}_i}{\partial v_{T+s}} + \sum_{i=1}^T \frac{\partial PVI}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial v_{T+s}} \quad (\text{B1})$$

First the focus will be on the second term of the sum in equation (B1). Combining equations (4.1) and (6) yields $u_\tau = \gamma \lambda_{\tau+1}$, and substituting this into equation (4), it is straightforward to check that $\frac{\partial PVI}{\partial \lambda_i}$ is the same for all v_{T+s} , $s \geq 0$:

$$\frac{\partial PVI}{\partial \lambda_i} = (1+r)^{-i} \gamma \lambda_i \quad (\text{B2})$$

Using equation (A9) from appendix A:

$$\frac{\partial \lambda_i}{\partial v_{T+s}} = p(\alpha + n\beta L_T)(1+r)^{-(T+s-i)} = (1+r)^{-s} \frac{\partial \lambda_i}{\partial v_T} \quad \text{for } 1 \leq i < T \quad (\text{B3})$$

where L_T is the (constant) value of labor at one of the long run equilibria.

Now turning to the first term of the sum in equation (B1). Equation (22) can be rewritten as:

$$L_\tau - \gamma \sum_{j=1}^{\tau-1} (1+r)^j \left[\sum_{i=j}^{T-1} \frac{d_i^v}{(1+r)^i} \right] = L_0 + \gamma \sum_{j=1}^{\tau-1} (1+r)^j \left[\sum_{i=T}^{\infty} \frac{d_i^v}{(1+r)^i} \right] \quad (\text{B4})$$

for $1 \leq \tau < T$.

Equation (B4) represents a system of $T-1$ linear equations, which can be written in matrix form as $A\vec{L} = \vec{b}$:

$$\begin{pmatrix} a_{11}(v_1) & \cdots & a_{1,T-1}(v_{T-1}) \\ \vdots & \ddots & \vdots \\ a_{T-1,1}(v_1) & \cdots & a_{T-1,T-1}(v_{T-1}) \end{pmatrix} \begin{pmatrix} L_1 \\ \vdots \\ L_{T-1} \end{pmatrix} = \begin{pmatrix} b_1(\{v_i\}_1^\infty) \\ \vdots \\ b_{T-1}(\{v_i\}_1^\infty) \end{pmatrix} \quad (\text{B5})$$

where $b_i = L_0 + K_i \left(\{v_k\}_1^{T-1} \right) + \gamma(1+r)^i \sum_{k=T}^{\infty} \frac{d_k}{(1+r)^k}$, K_i is a function of tariffs previous to

period T , $a_{ii} = 1 - \frac{\gamma p \beta v_i}{(1+r)^i} \left(\sum_{k=1}^{i-1} (1+r)^k \right)$, and $a_{ij} = -\frac{\gamma p \beta v_j}{(1+r)^j} \left(\sum_{k=1}^{\min\{i,j-1\}} (1+r)^k \right)$.

For this system of equations to have a solution, matrix A has to be non-singular. The system was constructed imposing the condition that a specific long run equilibrium will be reached at period T . If matrix A turns out to be singular for any tariff schedule, it means the equilibrium is not achievable in that time frame from the initial position. Limiting the values for T as suggested in section 3.1 overcomes this problem.

Using Cramer's rule to solve for the value of each labor allocation, we have:

$$L_\tau = \frac{D(A_\tau^{\bar{b}})}{D(A)} \quad (\text{B6})$$

where $D(A)$ represents the determinant of matrix A , and $A_\tau^{\bar{b}}$ is a matrix obtained from A by replacing its τ^{th} column by the vector \bar{b} .

We want to know the value of the derivative $\frac{\partial L_\tau}{\partial v_{T+s}} \forall s \geq 0$. The denominator of the function that determines L_τ (equation (B6)) does not depend on the value of the tariffs after time T . Therefore:

$$\frac{\partial L_\tau}{\partial v_{T+s}} = \frac{1}{D(A)} \frac{\partial D(A_\tau^{\bar{b}})}{\partial v_{T+s}} \quad (\text{B7})$$

$D(A_\tau^{\bar{b}})$ can be expanded using the τ^{th} column, yielding:

$$D(A_\tau^{\bar{b}}) = (-1)^{1+\tau} b_1 D(A_{1\tau}) + \dots + (-1)^{(T-1)+\tau} b_{T-1} D(A_{T-1,\tau})$$

where $A_{i\tau}$ is a matrix obtained from $A_{\tau}^{\bar{b}}$ by excluding row i and column τ from it.

Now equation (B7) can be rewritten as:

$$\frac{\partial \mathcal{L}_{\tau}}{\partial v_{T+s}} = \frac{\frac{\partial b_1}{\partial v_{T+s}} (-1)^{1+\tau} D(A_{1\tau}) + \dots + \frac{\partial b_{T-1}}{\partial v_{T+s}} (-1)^{(T-1)+\tau} D(A_{T-1,\tau})}{D(A)} \quad (\text{B8})$$

We know that:

$$\frac{\partial b_i}{\partial v_{T+s}} = \gamma (1+r)^i \frac{p(\alpha + n\beta L_T)}{(1+r)^{T+s}} = (1+r)^{-s} \frac{\partial b_i}{\partial v_T} \quad (\text{B9})$$

Combining equations (B8) and (B9), we finally get:

$$\frac{\partial \mathcal{L}_{\tau}}{\partial v_{T+s}} = (1+r)^{-s} \frac{\partial \mathcal{L}_{\tau}}{\partial v_T} \quad (\text{B10})$$

Substituting equation (B3) and (B10) into (B1) the proof is finished:

$$\frac{\partial PVI}{\partial v_{T+s}} = \sum_{i=1}^T \frac{\partial PVI}{\partial L_i} (1+r)^{-s} \frac{\partial \mathcal{L}_{\tau}}{\partial v_T} + \sum_{i=1}^T \frac{\partial PVI}{\partial \lambda_i} (1+r)^{-s} \frac{\partial \lambda_{\tau}}{\partial v_T} = (1+r)^{-s} \frac{\partial PVI}{\partial v_T} \quad (\text{B11})$$

Appendix C

The problem for the policy maker after the long run equilibrium is reached is to maximize the indirect utility function from that moment on. First the function must be derived for this time frame.

In period T consumers will maximize:

$$\sum_{\tau=T}^{\infty} \frac{1}{(1+\theta)^\tau} [a \log C_\tau^N + b \log C_\tau^A] \quad (C1)$$

subject to the following budget constraints, observing that the quantity of assets held at the beginning of period T is exogenously given from that period's point of view:

$$\begin{aligned} f_{\tau+1} - f_\tau &= r f_\tau + I_\tau^p - p(1+v_\tau)C_\tau^N - C_\tau^A + R_\tau, \quad \tau \geq T \\ f_T &= \sum_{i=0}^T (1+r)^{T-i} [I_i^p - p(1+v_i)C_i^N - C_i^A + R_i] \end{aligned} \quad (C2)$$

The first order conditions for the maximization above are analogous to those derived previously, i.e., equations (17.a-d); and the transversality condition in equation (18) can be rewritten as:

$$\lim_{\tau \rightarrow \infty} \frac{y}{(1+\theta)^\tau} f_{T+\tau} = 0 \quad (C3)$$

Using equation (17.d) and the initial value for f_T , it follows that:

$$f_{T+\tau} = (1+r)^\tau f_T + \sum_{i=0}^{\tau} (1+r)^{\tau-i} [I_{T+i}^p - p(1+v_{T+i})C_{T+i}^N - C_{T+i}^A + R_{T+i}] \quad (C4)$$

Using the government's budget constraint, and substituting equations (17.a) and (17.b) into equation (C4), the transversality condition yields:

$$y = \frac{S_T}{PVI_T + \phi_T} \quad (C5)$$

where $S_T = \sum_{i=0}^{\infty} (1+r)^{-i} \left[\frac{a}{(1+v_{T+i})} + b \right]$, $\phi_T = \sum_{i=0}^T (1+r)^{T-i} [I_i - pC_i^N - C_i^N]$ and

$$PVI_T = \sum_{i=0}^{\infty} (1+r)^{-i} I_{T+i} .$$

Substituting equation (C5) into (17.a) and (17.b), and then substituting these optimal consumption decisions into the utility function, give the following indirect utility function from time T on:

$$V_T(L_\tau, v_\tau) = \bar{K} - \frac{(1+\theta)(a+b)}{\theta} \log S_T - a \sum_{i=T}^{\infty} (1+\theta)^{T-i} \log(1+v_i), \quad (C6)$$

where \bar{K} is a constant term.

Note that PVI_T is now included in the constant term due to the fact that the labor allocation is constant from period T on. $PVI_T = I^E \frac{(1+r)}{r}$, where I^E is the per period income at the equilibrium point.

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